

Theoretical status of the lifetime predictions:

$$(\Delta\Gamma/\Gamma)_{B_s}, \tau_{B^+}/\tau_{B_d} \text{ and } \tau_{\Lambda_b}/\tau_{B_d}$$

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We give a review of the theoretical status of the lifetime predictions in the standard model. In case of $(\Delta\Gamma/\Gamma)_{B_s}$ we are already in a rather advanced stage. We obtain $(\Delta\Gamma/\Gamma)_{B_s} = (9.3^{+3.4}_{-4.6})\%$. It seems to be difficult to improve these errors substantially. In addition now some experimental results are available. For τ_{B^+}/τ_{B_d} and $\tau_{\Lambda_b}/\tau_{B_d}$ the theoretical status is much less advanced and the discrepancy between experiment and theory still remains. We conclude with a what-to-do-list for theorists.

1 Phenomenology

Inclusive decays of B hadrons are expected to be theoretically very clean. Therefore they probe an ideal testing ground for our understanding of QCD in weak decays. Unfortunately there are some discrepancies between experiment and theory, not dramatic, which still have to be resolved. The determination of the semileptonic branching ratio still does not agree perfectly with the experimental number. Closely related to that problem is the so-called missing charm puzzle^{1,2}. If one tunes the input parameter to get a better agreement for n_c one obtains with the same parameters worse values for B_{sl} . The present status of the missing charm puzzle can be found in³. Here clearly more work has to be done. The biggest problem in inclusive decays is still the lifetime of the Λ_b baryon. In the following we discuss the theoretical status of lifetime predictions of B hadrons in the standard model, in particular the decay rate difference $(\Delta\Gamma/\Gamma)_{B_s}$ and the lifetime ratios τ_{B^+}/τ_{B_d} and $\tau_{\Lambda_b}/\tau_{B_d}$ ^a.

1.1 The decay rate difference $(\Delta\Gamma/\Gamma)_{B_s}$

Now first experimental numbers for the width difference in the B_s system are available from LEP and CDF⁵:

$$\left(\frac{\Delta\Gamma}{\Gamma}\right)_{B_s} = 0.24^{+0.15}_{-0.13} \text{ or } \left(\frac{\Delta\Gamma}{\Gamma}\right)_{B_s} < 0.52 \text{ at } 95\%C.L.$$

If one uses in the experimental analysis the theoretical motivated constraint $1/\Gamma_s = \tau_{B_d}$, one gets a central value of 16% with an error of $\pm 9\%$. This limits

^aA more detailed discussion can be found in⁴ and references therein.

are already very close to the theoretical expectation⁶. $\Delta\Gamma_{B_s}$ will be measured quite precisely in the near future at TeVatron⁷. Several factors contribute to the big interest in $\Delta\Gamma_{B_s}$: a large value of the width difference opens up the possibility for novel studies of CP violation without the need for tagging⁸. Moreover, an experimental value of $\Delta\Gamma_{B_s}$ would give information about the still unknown mass difference in the B_s system⁹^b. Another interesting point is that new physics can only lead to a decrease of the width difference compared to the standard model value¹¹. An experimental number which is considerably smaller than the theoretical lower bound, would thus be a hint for new physics that affects B_s - \bar{B}_s mixing^c. Besides the need for a reliable theoretical prediction of $\Delta\Gamma_{B_s}$ in order to fulfill the above physics program it is of conceptual interest to compare experiment and theory in order to test local quark-hadron duality, which is the underlying assumption in calculating heavy quark decay rates. One can show that duality holds exactly in the limit $\Lambda_{\text{QCD}} \ll m_b - 2m_c \ll m_b$ and $N_c \rightarrow \infty$ ¹³. So far no deviation from duality has been conclusively demonstrated experimentally and theoretical models of duality violation in B decays tend to predict rather small effects¹⁴.

1.2 The lifetime ratios: τ_{B^+}/τ_{B_d} and $\tau_{\Lambda_b}/\tau_{B_d}$

The lifetime ratios of the B-hadrons are quite well known experimentally¹⁵.

$$\frac{\tau_{B^+}}{\tau_{B_d}} = 1.074 \pm 0.028, \quad \frac{\tau_{B_s}}{\tau_{B_d}} = 0.951 \pm 0.037, \quad \frac{\tau_{\Lambda_b}}{\tau_{B_d}} = 0.784 \pm 0.033.$$

BaBar and Belle expect to pin down the error of the first ratio to $\pm 1\%$ ¹⁶, BaBar has already a new result: 1.082 ± 0.028 ¹⁷. The lifetimes of the heavier B_s meson and the Λ_b hadron will be measured at CDF⁷. Theoretically one expects all these ratios to be very close to unity, which is clearly not the case.

2 Theoretical status

In the framework of the Heavy Quark Expansion (HQE)^d one can expand the decay rate in inverse powers of the heavy quark mass

$$\Gamma = \Gamma_0 + \left(\frac{\Lambda}{m_b}\right)^2 \Gamma_2 + \left(\frac{\Lambda}{m_b}\right)^3 \Gamma_3 + \left(\frac{\Lambda}{m_b}\right)^4 \Gamma_4 + \dots$$

The leading term is described by the decay of a free quark and the first non-perturbative corrections arise at the second order in the expansion. Γ_0 and

^bThe experimental status of ΔM_s is reviewed in¹⁰.

^cA detailed discussion about new physics effects in the B_s system can be found in¹².

^dFor a review see e.g.¹⁸.

Γ_2 cancel out in the decay rate difference and in the lifetime ratios (except for Λ_b) and will therefore not be discussed in more detail. In the third order we get the so-called weak annihilation and pauli interference diagrams. Here the spectator quark is included for the first time. These diagrams give rise to different lifetimes for different B hadrons. Schematically one can write the Γ_i 's as products of perturbatively calculable functions (Wilson coefficients) and matrix elements, which have to be determined by some non-perturbative methods like lattice QCD or sum rules.

2.1 The decay rate difference $(\Delta\Gamma/\Gamma)_{B_s}$

The decay rate difference can be written in the following way

$$\Delta\Gamma = \frac{\Lambda^3}{m_b^3} \left[\left(\Gamma_3^{(0)} + \frac{\alpha_s}{4\pi} \Gamma_3^{(1)} + \dots \right) + \frac{\Lambda}{m_b} \left(\Gamma_4^{(0)} + \dots + \right) + \dots \right].$$

$\Gamma_3^{(0)}$ was calculated in¹⁹ long time ago, the $1/m_b$ corrections were done in $\Gamma_4^{(0)}$ ²⁰ and the NLO QCD calculation was presented in $\Gamma_3^{(1)}$ ²¹. Both corrections gave a large reduction of the theoretical result. Meanwhile many unquenched ($N_f = 2$) lattice calculations of the non-perturbative constants (f_{B_s} , B and B_S) which appear in Γ_3 were done in^{22,23,24}.^e The non-perturbative parameters which enter Γ_4 are still unknown. In²⁰ the effect of these parameters was estimated in vacuum insertion approximation.

The improvement in theory input motivated the update⁶ of the result for $\Delta\Gamma_{B_s}$ presented in²¹. We also clarified the origin of seemingly disagreeing recent evaluations of $\Delta\Gamma_{B_s}$: The authors of^{23,24} were introducing a different normalization of $\Delta\Gamma/\Gamma$ in order to get rid of the dependence on $f_{B_s}^2$ with the price of getting a strong dependence on the relatively unknown CKM-parameter $|V_{ts}/V_{td}|$. With that method one gets a central value for $(\Delta\Gamma/\Gamma)_{B_s}$ of about 5%. We^{21,6} were normalizing the relative decay width difference to the semileptonic branching ratio, which should be theoretically very well understood^f. As lattice calculations are improving the dependence on $f_{B_s}^2$ should be less important in future. With the values $f_{B_s} = (230 \pm 30)$ MeV, $B(m_b) = 0.9 \pm 0.1$ and $\bar{B}_S = 1.25 \pm 0.1$ we get as a final number

$$\left(\frac{\Delta\Gamma}{\Gamma} \right)_{B_s} = (9.3_{-4.6}^{+3.4}) \%$$

^eThe advanced stage of lattice calculations can be read off from a comparison of lattice results for $f_{B_d}^2 B_{B_d}$ with fits of these parameters from the unitarity triangle²⁵.

^fIn principle there could be a similar problem as in the missing charm puzzle³, therefore we need precise experimental determinations of B_{sl} and n_c !

which coincides with the most recent determination in ²².

2.2 The lifetime ratios: τ_{B^+}/τ_{B_d} and $\tau_{\Lambda_b}/\tau_{B_d}$

The lifetime ratio of two B mesons can be written in the following way ⁹

$$\frac{\tau_1}{\tau_2} = 1 + \frac{\Lambda^3}{m_b^3} \left[\left(\Gamma_3^{(0)} + \frac{\alpha_s}{4\pi} \Gamma_3^{(1)} + \dots \right) + \frac{\Lambda}{m_b} \left(\Gamma_4^{(0)} + \dots \right) + \dots \right] .$$

Here the situation is very different. Only $\Gamma_3^{(0)}$ ^{26,2} is known. NLO corrections, both in $1/m_b$ and α_s still are missing. The non-perturbative matrix elements in Γ_3 for the meson lifetime ratio were calculated with QCD sum rules ^{27,28} and in quenched lattice QCD ²⁹. For the Λ_b hadron only preliminary lattice studies are available ³⁰. In a recent review ³¹ the following theoretical numbers were given

$$\frac{\tau_{B^+}}{\tau_{B_d}} = 1.03 \pm 0.04 , \quad \frac{\tau_{\Lambda_b}}{\tau_{B_d}} = 0.92 \pm 0.02 .$$

This result shows that the leading $1/m_b^3$ corrections to τ_{Λ_b} are sizeable. From $\Delta\Gamma_{B_s}$ we have learnt, that $\mathcal{O}(\alpha_s)$ and $\mathcal{O}(1/m_b)$ corrections to $\Gamma_3^{(0)}$ are important. So we still have to wait, till we can claim the discovery of new physics in the lifetime of the Λ_b baryon.

3 Prospects for improvement

For the lifetime ratios τ_{B^+}/τ_{B_d} and $\tau_{\Lambda_b}/\tau_{B_d}$ the next steps are clear: the calculation of the NLO QCD corrections ($\Gamma_3^{(1)}$) and the $1/m_b$ corrections ($\Gamma_4^{(0)}$) has to be finished. In the case of $\Delta\Gamma_{B_s}$ all these corrections were quite sizeable (about 50% of the LO result!). For the Λ_b baryon a lattice determination of the non-perturbative matrix elements in Γ_3 is still missing and for mesons one would like to have unquenched results, too.

The next class of improvements (both for $\Delta\Gamma$ and the lifetime ratios) could consist of the determination of the non-perturbative matrix elements of the dimension 7 operators, which appear in Γ_4 . Moreover one could do NLO QCD corrections to the $1/m_b$ corrections ($\Gamma_4^{(1)}$). As $\Gamma_4^{(0)}$ with vacuum insertion approximation for the bag parameters is very sizeable in the case of $\Delta\Gamma_{B_s}$ ²⁰, this effort could be really worth doing it. Of course, the errors for the decay constants and the bag parameter will become smaller in future lattice simulations. To clarify definitely the problem of getting different results for

⁹For the ratio $\tau_{\Lambda_b}/\tau_{B_d}$ we have an additional $1/m_b^2$ term, which is only known in leading order QCD.

$\Delta\Gamma/\Gamma$ from different normalizations it would be very helpful to have precise numbers for B_{sl} from experiment.

Finally, there are some more possibilities which seem to be quite improbable, to be done in the near future: the NLO QCD corrections to Γ_2 for Λ_b and the NNLO QCD corrections to Γ_3 , which would reduce the sizeable μ -dependence of $\Gamma_3^{(1)}$.

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